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STRUCTURAL SYNTHESIS OF SANDWICH BEAMS WITH OUTER LAYERS OF BOX-SECTION

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It is proved by model measurements that, for sandwich beams constructed from two rectangular tubes and a damping layer glued between them, the following calculation methods can be applied. Static bending and shear stresses as well as deflections of simply supported beams may be calculated by Allen's formulae for sandwich beams with flexurally stiff faces. The first eigenfrequency and the loss factor can be determined by using the diagrams given in reference [1]. For the loss factors Ungar's formula gives a suitable approximation. A minimum cost design procedure is presented for a sandwich beam with constant cross-section. The unknown dimensions of the cross-section are determined which satisfy the design constraints and minimize the material costs. In a numerical example, constraints relating to the maximal dynamic stresses and deflection as well as local buckling of plate elements are considered. In the optimization the backtrack combinatorial discrete programming method is applied. A numerical comparison shows that the material costs of a sandwich beam are lower than those of a homogeneous box one.

1. INTRODUCTION

The vibration damping of welded metal structures is very low. Special welded connections may improve their damping capacity [2], but a significant improvement can be achieved only by using sandwich structures with damping layers.

Sandwich panels with thin faces are not stiff enough; therefore it is advantageous to use flexurally stiff faces. Figure 1 shows some cross-sections of sandwich beams with

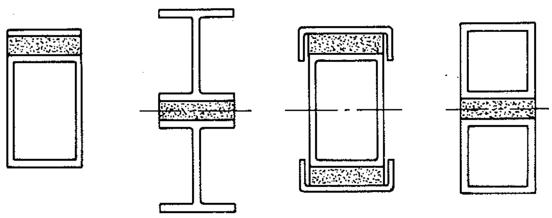

Figure 1. Cross-sections of sandwich beams with one or more flexurally stiff faces.

flexurally stiff outer layers. Yin, Kelly and Barry have shown [3] that a relatively high damping can be achieved by sandwich beams constructed from two rectangular thin-walled tubes and a constrained damping layer glued between them (see Figure 2(c)).

This paper is concerned with the static and dynamic analysis as well as minimum cost design of sandwich beams consisting of two tubes and a damping layer between them [4].

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2. STATIC ANALYSIS

Three model beams were tested. The beam sizes are shown in Figure 2. Square aluminium tubes and rubber cores were used. Two (A and B) sorts of rubber were used with densities of 1·25 and 1·05 g/cm^3, respectively.

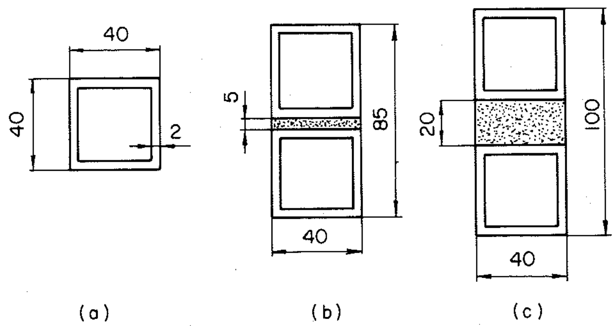

Figure 2. Dimensions of model beams tested (in mm). Span length of simply supported beams: $l = 1500$ mm.

The measured values of the static shear modulus G_s for the rubber materials A and B were 2·36 and 1·14 N/mm^2, respectively. On the basis of the maximal bending stresses and maximal deflection measured on the model shown in Figure 2(a) it was determined that the material of the square tubes has a modulus of elasticity of $E = 7 \times 10^4$ N/mm^2 ($G = 2{\cdot}63 \times 10^4$ N/mm^2). Hooke's law is valid for stresses below $\sigma_a = 110$ N/mm^2.

The static behaviour of sandwich beams with flexurally stiff faces can be analyzed on the basis of Allen's paper [5] with the following assumptions: (1) the stiffness of the core may be neglected, i.e., normal stresses do not occur in core and the shear stresses in a core cross-section are constant; (2) the transverse strain in the core can be neglected; (3) the shear deformations of the outer layers may be neglected.

In the case of a sandwich beam shown in Figure 3 the maximal deflection is given by (a list of symbols is given in the Appendix)

$$w_{\max} = (Fl^3/48B) + (Fl/4B_q)(1 - B_f/B)[1 - (\tanh \chi)/\chi], \tag{1}$$

$$\chi = \tfrac{1}{2}[(B_f/B_q l^2)(1 - B_f/B)]^{-1/2}, \tag{2}$$

$$B = B_f + B_s, \qquad B_f = 2E_1 I_1, \qquad B_s = E_1 A_1 d^2/2, \tag{3}$$

$$B_q = G_s b d^2/h_2, \qquad I_1 = \tfrac{1}{6} t_1 (h_1 - 2t_1)^3 + \tfrac{1}{2} b t_1 h_1^2, \qquad A_1 = 2t_1(h_1 - 2t_1) + 2bt_1. \tag{4–6}$$

The maximal shear stress in the core can be expressed as

$$\tau_2 = [F/2b(h_1 + h_2)](1 - B_f/B)(1 - 1/\cosh \chi). \tag{7}$$

The maximal normal stress is given by

$$\sigma_{1\max} = (E_1 Fl/4)\{(1/B)(h_1 + h_2/2)[1 - (\tanh \chi)/\chi] + h_1(\tanh \chi)/2B_f \chi\}. \tag{8}$$

Formulae have also been given for a uniformly distributed load by Allen [5] and for two concentrated forces by Grosskopf and Winkler [6].

Diagrams of the normal and shear stresses are shown in Figure 3. From the equilibrium of a quarter of the beam it can be seen that the following stress components occur:

$$\sigma' = F/8W_1, \qquad \sigma'' = \tau_2 bl/2A_1, \qquad \sigma''' = \tau_2 blh_1/4W_1. \tag{9}$$

Here $W_1 = 2I_1/h_1$.

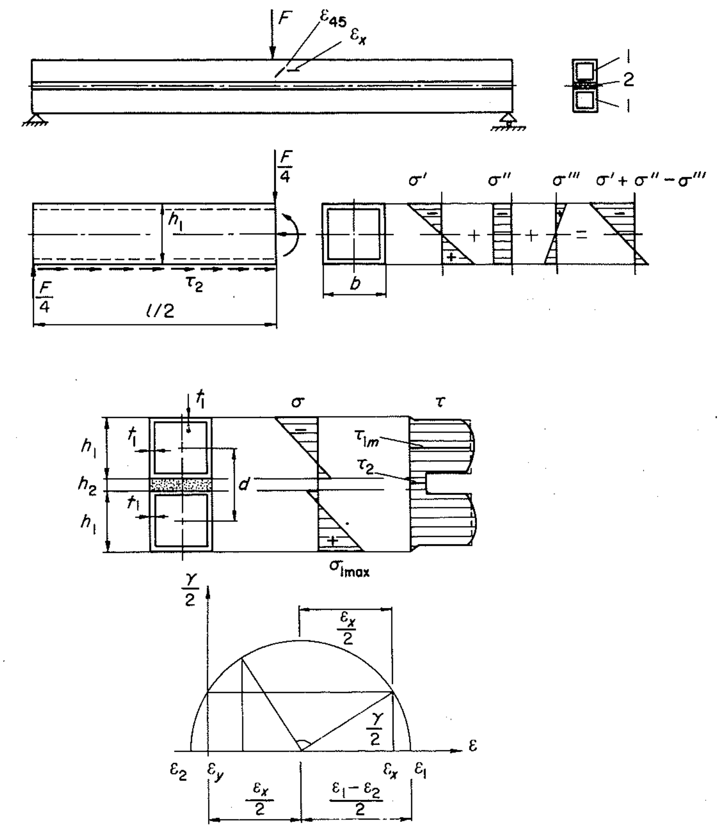

Figure 3. Stress diagrams for a symmetrical sandwich beam: σ' from bending caused by forces $F/4$; σ'' and σ''' from compression and bending due to the resultant force of shear stresses τ_2, respectively.

The mean shear stress in the webs of the outer layer tubes can be approximately calculated as

$$\tau_{1m} = [(F/2) - \tau_2 b h_2]/4t_1(h_1 - 2t_1). \tag{10}$$

In the experimental determination of the shear stress distribution measuring gauges were used in the 45° direction as shown in Figure 3. For the model beam (b) loaded with a concentrated force of $F = 2360$ N the following ε-values were measured: $\varepsilon_x = 280 \times 10^{-6}$; $\varepsilon_y = 0$; $\varepsilon_{45} = 56 \times 10^{-6}$. By means of Mohr's circle shown in Figure 3 the main strains ε_1 and ε_2 as well as the angular distortion γ may be calculated as follows:

$$\varepsilon_{1,2} = [(\varepsilon_x + \varepsilon_y)/2] \pm (\sqrt{2}/2)\sqrt{(\varepsilon_x - \varepsilon_{45})^2 + (\varepsilon_{45} - \varepsilon_y)^2}, \tag{11}$$

$$\gamma/2 = \sqrt{[(\varepsilon_1 - \varepsilon_2)/2]^2 - (\varepsilon_x/2)^2}. \tag{12}$$

From these formulae one obtains $\varepsilon_1 = 303 \times 10^{-6}$, $\varepsilon_2 = -23 \times 10^{-6}$, $\gamma/2 = 83 \times 10^{-6}$ and $\tau_{1\max} = G\gamma = 4{\cdot}36\ \mathrm{N/mm^2}$.

In the calculations the following data were used (dimensions in mm): $l = 1500$, $b = h_1 = 40$, $h_2 = 5$, $t_1 = 2$. By using formulae (1)–(8) one gets $A_1 = 308\ \mathrm{mm^2}$, $I_1 = 7{\cdot}9605 \times 10^4\ \mathrm{mm^4}$, $B_f = 1{\cdot}1145 \times 10^{10}\ \mathrm{N\,mm^2}$, $B_s = 2{\cdot}1830 \times 10^{10}\ \mathrm{N\,mm^2}$, $B = 3{\cdot}2975 \times 10^{10}\ \mathrm{N\,mm^2}$, $B_q = 0{\cdot}8215 \times 10^6\ \mathrm{N\,mm^2}$, and $\chi = 1{\cdot}7091$.

The measured and calculated values of stresses and deflection given in Table 1 show a suitable agreement.

TABLE 1

Measured and calculated static stresses and deflection of a model beam (stresses in N/mm^2)

	Measured	Calculated
σ_{1max}	94·2	97·0
τ_{1max}	4·36	4·02
τ_2	—	0·282
w_{max}(mm)	10·5	9·61

3. DYNAMIC ANALYSIS

The dynamic characteristics of the core materials were measured by using the method proposed by Jones and Parin [7]. Based on the measured acceleration values at points P and Q, respectively (see Figure 4), the transmissibility at the resonance frequency is

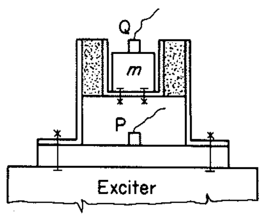

Figure 4. Measurement of dynamic characteristics of a core material.

given by

$$T_{PQ} = |\ddot{x}_Q/\ddot{x}_P| \tag{13}$$

and the loss factor of the core material can be calculated as

$$\eta_2 = (T_{PQ}^2 - 1)^{-1/2}. \tag{14}$$

The dynamic shear modulus may be expressed as

$$G_d = m\omega_1^2/2b_0, \tag{15}$$

where ω_1 is the measured eigenfrequency and b_0 is the width of the test specimen. η_2 and G_d-values of the rubber material A are given in Table 2 ($m = 2{\cdot}69$ kg, $b_0 = 140$ mm). Table 2 shows the calculated mean shear stresses as well.

The forced vibratory motion of a sandwich beam can be described by the following equation derived by Mead and Markuš [8]:

$$\partial^6 w/\partial x^6 - g_0(1+Y)\partial^4 w/\partial x^4 = (1/B_f)(\partial^4 p_0/\partial x^4 - g_0 p_0), \tag{16}$$

where $p_0 = -m\partial^2 w/\partial t^2 + p(x, t)$. For symmetrical sandwich beams

$$g_0 = 2G_d b/A_1 E h_2, \qquad Y = [(h_1 + h_2)^2/2B_f]A_1 E. \tag{17, 18}$$

Although equation (16) was derived for sandwich beams with thin faces, it will be shown that it can be applied for flexurally stiff faces as well.

TABLE 2

Measured and calculated dynamic characteristics. Specimen with rubber A (stresses in N/mm^2)

$\ddot{x}_P/g$	$\ddot{x}_Q/g$	f_1(Hz)	η_2	G_d(N/mm^2)	τ
1·0	2·5	157	0·095	9·34	0·0249
1·5	8·0	151	0·134	8·64	0·0265
2·0	9·5	150	0·151	8·52	0·0315
2·5	11·5	145	0·156	7·97	0·0382
3·0	13·5	144	0·166	7·80	0·0448
4·0	16·6	137	0·173	7·11	0·0551

For the calculation of the eigenfrequencies of a cantilever sandwich beam the diagrams given in the paper of Markuš and Valášková [1] may be used. The eigenvalues,

$$\lambda_n^4 = (mL^4/B_f)\omega_n^2, \tag{19}$$

are graphically given as functions of the parameters g_0L^2 and Y, respectively.

In their paper [1] the loss factors are also given graphically; however, the following formula of Ungar [9] is a suitable approximation:

$$\eta = \eta_2 XY/[1+(2+Y)X+(1+Y)(1+\eta_2^2)X^2], \tag{20}$$

$$X = g_0 r_k^2, \qquad r_k = Cl/2\pi. \tag{21}$$

X is the shear parameter. Values of the constant C have been given in reference [3] for various boundary conditions. For instance, in the case of a cantilever beam with unrestrained ends, for the first mode, $C = 1{\cdot}1$, and for a simply supported beam $C = 2{\cdot}0$.

In the following the results of measurements are compared with calculated values only for the sandwich beam for which the data have been given in section 2. The dynamic measurements were carried out on sandwich cantilevers by means of Brüel and Kjaer instruments as illustrated in Figure 5. The measured first eigenfrequency was f_1(measured) = 110 Hz.

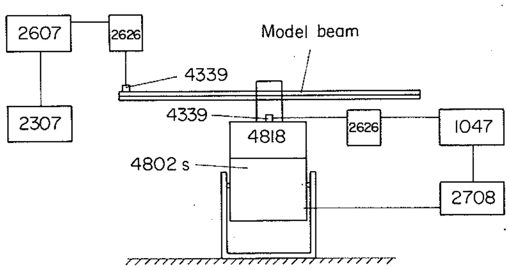

Figure 5. Dynamic measurements on a model beam with the following Brüel and Kjaer measuring instruments: 4802 S, exciter body; 4818, exciter head; 4339, accelerometer; 2626, conditioning amplifier; 1047, exciter control; 2708, power amplifier; 2607, low-noise measuring amplifier; 2307, level recorder.

By means of measurements of accelerations, the maximal dynamic force has been determined as $F_{max} = 179{\cdot}6$ N. With strain gauges as shown in Figure 3, the shear stress was also measured as $\tau_1 = 0{\cdot}584$ N/mm^2. By using equation (10) the shear stress in the core can be calculated:

$$\tau_2 = [F_{max} - 4t_1(h_1 - 2t_1)\tau_1]/bh_2 = 0{\cdot}057 \text{ N/mm}^2.$$

For this value the extrapolation of values given in Table 2 results in $G_d = 7\ \mathrm{N/mm^2}$ and $\eta_2 = 0{\cdot}18$. With data of $m = A\rho = 1{\cdot}9132 \times 10^{-6}\ \mathrm{N\,s^2 mm^{-2}}$, $B_f = 1{\cdot}1145 \times 10^{10}\ \mathrm{N\,mm^2}$ and $L = 74{\cdot}5$ mm, formula (19) gives $f_n = \omega_n/2\pi = 21{\cdot}886\lambda_n^2$.

On the basis of equations (17) and (18) one obtains $g_0 = 5{\cdot}1948 \times 10^{-4}\ \mathrm{mm^{-2}}$ ($g_0 L^2 = 2{\cdot}88$) and $Y = 1{\cdot}96$. From the diagram of Figure 7 of reference [1] one obtains $\lambda_1^4 = 24{\cdot}2$ and f_1 (calculated) = 107·7 Hz.

Further, with $C = 1{\cdot}1$ and $l = 1500$ mm equation (21) gives $r_k = 26{\cdot}25$ and by using equation (20) one obtains $X = 0{\cdot}3582$ and η(calculated) = 0·045.

The loss factor, obtained by means of the evaluation of the measured frequency diagram, by using the half-power bandwidth method, is η(measured) = 0·044.

It can be seen that the agreement between the measured and calculated f_1 and η-values is suitable. Note that, in the case of second and third eigenfrequency, a similar suitable agreement was found. Thus, the above described calculation methods can be applied for flexurally stiff outer layers as well.

4. MINIMUM COST DESIGN

4.1. GENERAL ASPECTS

The optimality of a structure may be defined in various ways. For instance, Markuš, Oravský and Šimková [10] have searched for the optimal configuration (dimensions of the cross-section) of a sandwich beam having maximal loss factor in the case of given values of G_d and η_2 as well as for a given vibration mode (natural frequency).

In what follows here the most widely used general definition will be applied: an optimal structure satisfies the design constraints and its mass or cost is minimal. Thus, in the optimum design procedure, a beam configuration (cross-sectional dimensions) is sought which satisfies the design constraints and minimizes the cost function. In this procedure, not only the technical requirements but also the economic aspects may be considered.

These are three types of *design constraints*: constraints of strength, those of production and other (e.g., aesthetical) requirements. Strength constraints may be formulated on the basis of possible failures or other limit states, when the structure is unfit for use (excessive deformations, general or local instability, fatigue, excessive vibrations, etc.). Production or technological constraints are mainly related to the dimensions of a structure (e.g., prescription of minimal thicknesses).

In the mathematical formulation of optimum design problems, the objective (merit) function expresses usually the material and production costs.

The structural synthesis has three main phases as follows: (i) *analytical phase*: selection of materials, profiles, type of structure, production technology, formulation of the design constraints and the cost function; (ii) *mathematical phase*: minimization of the objective function with fulfilment of the constraints; (iii) *evaluation, elaboration of results*: design aids, sensitivity analysis, etc.

The optimum design of simpler metal structures, e.g., sandwich beams of constant cross-section, is characterized by (1) a small number of unknowns; (2) unknowns of mostly cross sectional dimensions (plate thicknesses) to be chosen from a given list of discrete values determined by the production; (3) a cost function which is monotonically increasing; (4) a lot of intricate non-linear design constraints to be satisfied.

For solving optimum design problems a number of mathematical methods are known (see, e.g., reference [11]). In our research work we have successfully used the *backtrack programming method* suitable for problems characterized above. This combinatorial discrete programming method solves non-linear constrained function minimization problems by a systematic search procedure. A partial search is carried out for each variable and, if the possibilities are exhausted, a backtrack and a new partial search is performed. A substantial search reduction may be achieved by utilizing the fact that the objective function is monotonically increasing. A more detailed description of the method can be found, e.g., in reference [12], or [13].

4.2. NUMERICAL EXAMPLE

Consider a simply supported sandwich beam of span length $l = 3$ m, constructed from two aluminium alloy tubes and a rubber layer glued between them.

The objective function may be expressed as the material cost

$$K_m = 2A_1lk_1 + bh_2lk_2 + 2blk_3, \tag{22}$$

where k_1, k_2 and k_3 are material cost factors for an outer-layer profile of cross-sectional area A_1, damping layer and adhesive, respectively. Here the values $k_1 = 4400\ \$/\text{m}^3$, $k_2 = 1000\ \$/\text{m}^3$ and $k_3 = 20\ \$/\text{m}^2$ have been used.

The following design constraints should be considered.

1. *Constraint of the normal stress* in the extreme fibre of outer layers:

$$\sigma_{1\max} \leqslant \sigma_1^*, \tag{23}$$

where $\sigma_{1\max}$ is calculated with the aid of equation (8), $\sigma_1^* = 100\ \text{N/mm}^2$ being the admissible stress. In the case of a forced vibration, according to equation (14), if $\eta \leqslant 0{\cdot}1$, the loss factor can be expressed as

$$\eta \approx 1/T_R = |x_1/x_2| = |F_1/F_2|, \tag{24}$$

where T_R is the transmissibility at the resonance frequency, F_1 is the applied force, F_2 is the dynamic force, and x_1 and x_2 are the corresponding displacements (see also reference [14]). Thus, $\sigma_{1\max}$ is to be calculated with $F_2 = F_1/\eta$. For the calculation $F_1 = 475$ N and η is given by formula (20). Further data used were $E_{al} = 7\times10^4\ \text{N/mm}^2$, and for the rubber $G_s = 2{\cdot}36\ \text{N/mm}^2$ and $G_d = 7{\cdot}0\ \text{N/mm}^2$, and $\eta_2 = 0{\cdot}18$.

2. *Constraint of the shear stress* in the core:

$$\tau_2 \leqslant \tau_2^*. \tag{25}$$

A formula for τ_2 is given in equation (7) (calculated with the dynamic force F_2); $\tau_2^* = 2{\cdot}5\ \text{N/mm}^2$ is the admissible shear stress.

3. *Deflection constraint*:

$$w_{\max} \leqslant w^*. \tag{26}$$

$w_{\max}$ is to be calculated from equation (1) with the dynamic force F_2; $w^* = 20$ mm is the admissible deflection.

4. *Local buckling constraint for the compressed flange* of the upper box section:

$$t_1/b \geqslant \delta. \tag{27}$$

For a thin-walled rectangular hollow beam subjected to bending, made of steel Fe 360 ($\sigma_{1s}^* = 200$, $E = 2{\cdot}1\times10^5\ \text{N/mm}^2$), according to Frieze [15], $\delta = 1/30$. For an aluminium-alloy and for a stress level of $\sigma_{1al} = 100\ \text{N/mm}^2$,

$$1/\delta = 30\sqrt{\sigma_{1s}^*/\sigma_{1al}^*}\sqrt{E_{al}/E_s} = 25. \tag{28}$$

5. *Local buckling constraint for webs* subjected to bending and compression:

$$t_1/h_1 \geqslant \beta. \tag{29}$$

According to Frieze [15], for a steel web subjected to bending $\beta = 1/145$. In the case of a steel web subjected to bending (σ_M), compression (σ_N) and shear (τ), the following approximate interactive formula may be used [16]:

$$1/\beta = 145\sqrt[4]{[(1+\sigma_N/\sigma_M)^2+3(\tau/\sigma_M)^2]/[1+173(\sigma_N/\sigma_M)^2+20(\tau/\sigma_M)^2]}. \qquad (30)$$

For the webs of the upper box section it can be assumed that $\sigma_N/\sigma_M = 1$ and $\tau = 0$. For an aluminium alloy, formula (30) is reduced similarly as (28):

$$\frac{1}{\beta} = 145\sqrt[4]{\tfrac{4}{174}}\sqrt{\tfrac{2}{3}} = 40. \qquad (31)$$

The lists of discrete values of variables are given in Table 3. Note that the series of discrete values used in the computations were special ones satisfying the relation $x_{i\max} - x_{i\min} = \Delta x_i 2^q$, where q is an integer. For such series the interval halving method can be applied in one-dimensional search procedures.

TABLE 3

Data of the series of discrete values of variables (sizes in mm)

x_i	$x_{i\min}$	$x_{i\max}$	Δx_i
h_1	40	200	10
h_2	5	50	10
b	40	200	10
t_1	2	10	1

Results of computations carried out by the Fortran IV program of the backtrack method are as follows: optimal dimensions in mm: $h_1 = 120$; $b = 50$; $t_1 = 3$; $h_2 = 15$. Further, $\eta = 0{\cdot}0535$, $F_2 = 8878$ N, $\sigma_{1\max} = 95$ N/mm^2, $\tau_2 = 0{\cdot}237$ N/mm^2, $w_{\max} = 17{\cdot}2$ mm (the deflection constraint was not active); $K_{m\min} = 37{\cdot}36$ \$.

In order to illustrate the *economy* of such sandwich beams, one can compare the above optimal beam with a homogeneous one, assuming that the material damping factor of a homogeneous aluminium-alloy box beam is $\eta_h = \eta/6$. In view of equation (24), the beam should be designed for a dynamic force $F = F_2\eta/\eta_h = 53268$ N, with stress and local buckling constraints. For web buckling one takes

$$1/\beta' = 145\sqrt{\sigma^*_{1s}/\sigma^*_{1al}}\sqrt{E_{al}/E_s} = 145\sqrt{\tfrac{2}{3}} = 118.$$

The maximal bending moment is $M = 53268 \times 3/4 = 39950$ N m, and the required section modulus is $W_0 = M/\sigma^*_{1el} = 3{\cdot}995 \times 10^5$ mm^3. By using formulae derived for the optimal sizes of a box beam subjected to bending (see, e.g., reference [16]) one obtains

$$h_{\text{opt}} = \sqrt[3]{0{\cdot}75\,W_0/\beta'} = 328{\cdot}2 \text{ mm},$$

$t_w/2 = \beta' h_{\text{opt}} = 2{\cdot}78$ mm, $t_f = h_{\text{opt}}\sqrt{\beta'\delta} = 6$ mm. With rounded values of $h = 350$, $t_w/2 = 3$, $t_f = 6$ and $b_f = 135$ mm, one obtains $A = 3720$ mm^2. The material cost is $K'_m = 49{\cdot}10$ \$, which is 31% greater than that of the sandwich beam. The characteristics of the sandwich beam and the homogeneous box one are summarized in Figure 6 and Table 4.

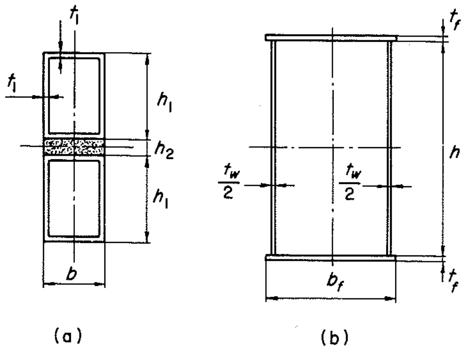

Figure 6. Optimal cross-sections (dimensions in mm). (a) Sandwich beam: $h_1 = 120$, $h_2 = 15$, $b = 50$, $t_1 = 3$; (b) homogeneous box beam: $h = 350$, $t_w/2 = 3$, $b = 135$, $t_f = 6$.

TABLE 4

Comparison of the characteristics of the two optimized beams

	Sandwich beam Figure 6(a)	Homogeneous box beam Figure 6(b)
Max. dynamic stress σ_{1max}(N/mm^2)	95·0	99·4
Max. dynamic deflection w_{max} (mm)	17·2	5·9
Loss factor η	0·0535	0·0089
Material costs ($)	37·36	49·10

REFERENCES

1. Š. MARKUŠ and C. VALÁŠKOVÁ 1972 *Journal of Sound and Vibration* **23**, 423–432. On eigenvalue boundary problems of transversely vibrating sandwich beams.
2. J. FARKAS 1973 *Schweisstechnik DDR* **23**, 508–512. Schwingungsdämpfung von Schweisskonstruktionen.
3. T. P. YIN, T. J. KELLY and J. E. BARRY 1967 *Journal of Engineering for Industry* **89**, 773–784. A quantitative evaluation of constrained-layer damping.
4. K. JÁRMAI 1979 *Ph.D. Thesis, Technical University of Miskolc.* Structural synthesis of sandwich beams with flexurally stiff faces (in Hungarian).
5. H. G. ALLEN 1973 in *Sheet Steel in Building, Papers and Discussions from the Meeting.* London: Iron and Steel Institute, pp. 10–18. Sandwich panels with thick or flexurally stiff faces.
6. P. GROSSKOPF and T. WINKLER 1973 *Kunststoffe* **63**, 881–888. Auslegung von GFK-Hartschaum-Verbundwerkstoffen.
7. D. I. G. JONES and M. L. PARIN 1972 *Journal of Sound and Vibration* **24**, 201–210. Technique for measuring damping properties of thin viscoelastic layers.
8. D. J. MEAD and Š. MARKUŠ 1969 *Journal of Sound and Vibration* **10**, 163–175. The forced vibration of a three-layer damped, sandwich beam with arbitrary boundary conditions.
9. E. E. UNGAR 1962 *Journal of the Acoustical Society of America* **38**, 1082–1089. Loss factors of viscoelastically damped beam structures.
10. Š. MARKUŠ, V. ORAVSKÝ and O. ŠIMKOVÁ 1974 *Acta Technica Československé Akademie Věd* **19**, 647–662. Philosophy of optimum design of damped sandwich beams.
11. S. S. RAO 1978 *Optimization. Theory and Applications.* New Delhi: Wiley Eastern Limited.
12. J. FARKAS 1980 *11th Congress of the International Association of Bridge and Structural Engineers Vienna. Final Report, Zürich,* 597–602. Optimum design of metal structures by backtrack programming.
13. J. FARKAS 1981 *Berichte* IX. *Internationaler Kongress über Anwendungen der Mathematik in den Ingenieurwissenschaften Weimar. Band* 1*.* 70–73. Anwendung der Backtrack-Programmierungsmethode auf die Optimierung von geschweissten Stabtragwerken.

14. J. C. SNOWDON 1968 *Vibration and Shock in Damped Mechanical Systems.* New York: Wiley.
15. P. A. FRIEZE 1980 in *Thin-walled Structures.* London: Granada, pp. 455–477. Behaviour and design of thin-walled rectangular hollow beams.
16. J. FARKAS 1978 *Acta Technica Academiae Scientiarum Hungaricae* **87**, 295–306. Minimization of the cross-section area of welded unstiffened plate and box girders subjected to bending and shear.

APPENDIX: LIST OF SYMBOLS

A	cross-sectional area
B, B_f, B_s	bending stiffnesses of a sandwich beam, equation (31)
B_q	shear stiffness of a sandwich beam, equation (4)
b	width
C	a constant, equation (21)
E_s, E_{al}	modulus of elasticity of steels and Al-alloys, respectively
F	force
f	eigenfrequency
G_s, G_d	static and dynamic shear modulus, respectively
g	gravitational acceleration
g_0	a parameter, equation (17)
h	height
I	modulus of inertia
K_m	material cost
k	cost coefficient
L, l	span length, length
M	bending moment
m	mass
p	intensity of a uniform normal load
r_k	a parameter, equation (21)
T_R, T_{PQ}	transmissibility
t	time
t_1	thickness of rectangular tubes
t_f	flange thickness
t_w	web thickness
W	section modulus
W_0	required section modulus
w	deflection
w^*	admissible deflection
X	shear parameter, equation (26)
x_i	variables
$\ddot{x}$	acceleration
Y	a parameter, equation (18)
β	limiting plate slenderness for webs
γ	angular distortion
δ	limiting plate slenderness for flanges
ε	specific strain
η	loss factor of the sandwich beam
η_2	loss factor of the damping layer
λ	eigenvalue, equation (19)
ρ	density
σ	normal stress
σ^*	admissible normal stress
τ	shear stress
τ^*	admissible shear stress
χ	sandwich beam parameter, equation (2)
ω	angular frequency